ISSN 2229-5518

Improve the linear programming model and solved using computer algorithms

Mobarak Abaker Adam Hassn

Abstract –This paper aims to improve the linear programming model ,and solved it using computer algorithms, by converting the linear programming model into a computer algorithms which to find the basic feasible solution and then search to improve it according to mathematical relationships and equations on the constraints of linear programming model

The paper concluded that the computer algorithms and improve solution help to find optimal solution quickly. And that optimal solution helps to make decision

We recommend the development of this algorithm to solve all models of linear programming

Keyword –Linear Programming Model, Optimization, Mathematical Programs, basic feasible solution

1. Introduction

Operations research is a scientific approach to decision-making related to business management. Operations research models have been accepted for application in business, industrial, agricultural and service institutions such as transportation and health. The most important of these is the linear programming models used to optimize allocation of resources Limited to alternative uses in a way that achieves a particular objective as best as possible.

2. Optimization problems

Managers, planner,etc., are repeatedly faced with complex and dynamic systems, which they have to manager or control in order to realize certain goal. These systems have the following properties in common:

- Decision variables representing the options and operating levels the decision-maker can control in order to drive the behavior of the systems
- Constraints limiting the range of control the decision-maker has on the decision variables
- An objective that measures how well the system is operating with regard to the goal of the decision maker.

- *Mobarak Abaker Adam Hassn is Assistants Professor in Computer Science in Shaqra University Sajir College, KSA / Alneelain University Facility of computer & information technology, SUDAN, PH-00966556717762. E-mail: mhassan@su.edu.sa*

3. Mathematical Programs

Mathematical programs are simply the presentation of an optimization problem in a mathematically precise from. The general description of a mathematical program is of the form

$$(MP)\begin{Bmatrix} max \\ min \end{Bmatrix}_{x\in X} f(x)$$

Where

- x is the set of decision variables, which can be represented by numerical values, sets, functions,etc.
- X represents the feasible region describing the constraints on the decision variables. Feasible regions are usually described by giving equalities and inequalities involving functions of the decision variables. Any point $x \in X$ is called a feasible solution to the problem.
- F represents the objective function, a function of the variable values that one wishes to maximize or minimize.

3.1 Solving Mathematical programs

The goal of solving a mathematical program (MP) is to find an optimal solution to the problem, that is an assignment x^* of values to the decision variables x in such a way that

i. x^* is feasible to (MP)
ii. $x^*(MP)$ has the " best" objective function value for (MP) in the sense that any other feasible solution x to (MP) has
$$f(x) \le f(x^*) \quad (for\ a\ max\ problem\ )$$
$$f(x) \ge f(x^*) \quad (for\ a\ min\ problem\ )$$

4. Linear Programming Models (LP Models)

ISSN 2229-5518

Each of the models above are examples of linear programs. Linear programming models common terminology for programming:

4.1 Linear programming models involve

- Resources denoted by i, there are m resources
- Activities denoted by j, there are n activities
- Performance measure denoted by z

An LP Models:

$$\max \quad z = \sum_{j=1}^{n} c_j\, x_j$$

s.to

$$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \quad \forall_i = 1, \cdots, m$$

$$x_j \geq 0 \qquad \forall_j = 1 \cdots m$$

Z: value of overall performance measure

x_j: Level of activity j (j=1 . . . n)

c_j: Performance measure coefficient for activity j

b_i : amount of resource i available (i=1 . . . m)

a_{ij} : Amount of resource i consumed by each unit of activity j

Decision variable:x_j

Parameters:c_j , a_{ij} , b_j

4.2 Stander form of linear programming models (S.FOF LPM)

A linear programming problem can be expressed in the following stander form:

$$\max z = c_1\, x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_n x_n$$

s.to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x_m + a_{m2}x_2 + \cdots + a_{mn}x_{mn} \leq b_m$$
$$x_j \geq 0 \qquad \forall_j = 1,2,3, \cdots, n$$

Where

Objective function: overall performance measurez = *c1x1+c2x2+c3x3+⋯+cnxn*

Constraints:

$$\sum_{i,j=1}^{m,n} a_{ij}x_j \leq \sum_{i=1}^{m} b_j \forall_{i,j} = 1 \cdots m, 1 \cdots n \; (functional\ constraint\ )$$

$$x_j \;\geq\; 0 \quad \forall_j = 1, \cdots, n \qquad (\,Nonnegativity\ constraint\ )$$

4.3 Variations in LP Model

An LP Model can have the following variations:

1. Objective function : minimization or maximization problem
2. Direction of constraints

$$\sum_{i,j=1}^{m,n} a_{ij}x_j \quad \leq \sum_{i=1}^{m} b_j \forall_{i,j} = 1 \cdots m, 1 \cdots n \quad (\,less\ than\ or\ equal\ to\ )$$

$$\sum_{i,j=1}^{m,n} a_{ij}x_j \geq \sum_{i=1}^{m} b_j \forall_{i,j} = 1 \cdots m, 1 \cdots n \quad (\,greater\ than\ or\ equal\ to\ )$$

$$\sum_{i,j=1}^{m,n} a_{ij}x_j \;=\; \sum_{i=1}^{m} b_j \forall_{i,j} = 1 \cdots m, 1 \cdots n \quad (\,equality\ )$$

3. Non-negativity constraints $x_j \;\geq\; 0 \quad \forall_j = 1, \cdots, n$

4.4 Terminology for solution of the LP Model

- Solution: any specification of value for the decision variable x_j is called a solution.
- Infeasible solution: a solution for which at least one constraint is violated.
- Feasible solution: a solution for which all of the constrains are satisfied.
- Optimal solution a feasible solution that has the most favorable value of the objective function.

$$maximization \;\Longrightarrow largest\ z$$

$$minimization \;\Longrightarrow smallest\ z$$

4.5 Solve Linear Programming Model

To solved linear programming model

$$\max z\ OR\ min = c_1\, x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_n x_n$$

s.to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x_m + a_{m2}x_2 + \cdots + a_{mn}x_{mn} \leq b_m$$
$$x_j \geq 0 \qquad \forall_j = 1,2,3, \cdots, n$$

ISSN 2229-5518

Let

$X = (x_1, x_2, x_3, \cdots, x_n)^T x_j \in X_N \implies x_j$ are the variables of the problem and are allowed to take on any set of real values that satisfy the constraints.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} a_{ij} \in A_{MN} \implies$$

a_{ij} are parameters of the problem , and provide the precise description of a particular instance of linear programming model that you wish to solve

$$C = [c_1 \quad c_2 \quad c_3 \quad \cdots \quad c_n] c_j \in C_N \implies c_j \, are \, the \, objective/cost / profit \, coefficients$$

$$b = (b_1 \quad b_2 \quad b_3 \quad \ldots \quad b_m)^T \quad b_i \in b_n \implies b_j \, are \, the \, right - hand - side/resource$$

On that assume the linear programming model can be solved by

4.5.1 The basic feasible solutions

From linear programming model

$$\max OR \min Z = CX$$

S.TO

$$AX = b$$

$$x_j \geq 0$$

- ✓ Let $A = [B \quad N] \implies B, N \in A$

  $B \equiv basic \, matrix \, invraiable$

  $N \equiv Non \, basic \, matrix \, invariable$

  $$A = [B \quad N] \qquad , [B \quad N] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- ✓ Let $X = (X_B \quad X_N)^T \implies X_B, X_N \in X$
  $X_B \equiv Basic \, feasible \, solution$,
  $X_N \equiv Non \, basic \, feasible \, solution$

- ✓ Let $C = [C_B \quad C_N]$

Recompense those Assumptions in linear programming model

$$z = [C_B \quad C_N] \begin{bmatrix} X_B \\ X_N \end{bmatrix}$$

S.TO

$$[B \quad N] \begin{bmatrix} X_B \\ X_N \end{bmatrix} \leq b$$

$$X \geq 0$$

Then

$$z = C_B X_B + C_N X_N$$

S.TO

$$BX_B + NX_N \leq b$$

$$X \geq 0$$

Let $X_N = 0 \implies Non \, basic \, feasible \, solution$

$BX_B < b \quad or \; BX_B = b$

$X_B = B^{-1} b \; --- (1) \quad basic \, feasible \, solution$

$z = C_B X_B + C_N * 0 \implies$

$z = C_B B^{-1} b --- (2) \, the \, value \, of \, objective \, function$

Transforming a General LP into Equality Form

Minimization problems: replace

$$\min z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_n x_n$$

With

$$\max z^- = -z = -c_1 x_1 - c_2 x_2 - c_3 x_3 - \cdots - c_n x_n$$

≤ Constraints: add a nonnegative slack variable indicating the difference between the LHS value and the b_i value specifically replace

$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \leq b_i$

With

$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n + x_{n+1} = b_i$

$x_{n+1} \geq 0$

≥ Constraints: Subtract the slack variable in that row specifically replace

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \geq b_i$$

With

$$a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n - x_{n+1} = b_i$$

$$x_{n+1} \geq 0$$

Unrestricted variable: replace unrestricted x_i by

$$x_i = x_i^+ - x_i^-, x_i^+ \geq 0, x_i^- \geq 0$$

International Journal of Scientific & Engineering Research Volume 9, Issue 1, January-2018 324
ISSN 2229-5518

Negative variable: replace $x_i \leq 0\ by, x_i^- = -x_i, x_i \geq 0$

Example: use the basic feasible solutions find the optimal solution

$$max\ Z = 3X_1 + 2X_2$$

S.TO

$$X_1 + X_2 \leq 3$$

$$X_1 + 2X_2 \leq 5$$

$$X_1, X_2 \geq 0$$

Transforming a General LP into Equality Form

$$max\ Z = 3X_1 + 2X_2 + 0X_3 + 0X_4$$

S.TO

$$X_1 + X_2 + X_3 = 3$$

$$X_1 + 2X_2 + X_4 = 5$$

$$X_1, X_2, X_3, X_4 \geq 0$$

The matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix},\ C = [3 \quad 2 \quad 0 \quad 0]\ , X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

The first feasible solution

$$Let\ X_B = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix}$$

$$X_B = B^{-1}b\ - - - (1) \quad basic\ feasible\ solution$$

$$z = C_B B^{-1} b - - - (2)\ the\ value\ of\ objective\ function$$

$$C_B = [0 \quad 0]\ , B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, X_B = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$X_3 = 3\ , X_4 = 5\ , X_1 = 0\ , X_2 = 0,$
$Z_1 = C_B B^{-1} b = [0 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

The second feasible solution

$X_B^2 = \begin{bmatrix} X_3 \\ X_1 \end{bmatrix}, C_B = [0 \quad 3] \quad , \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B^{-1} = \frac{1}{|Adj\ B|} B^{\backslash} \quad ,$
$Adj\ B = 1$

$$a_{11} = 1\ , a_{12} = 0\ , a_{21} = -1\ , a_{22} = 1$$

$$B^{\backslash} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, B^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$X_B^2 = \begin{bmatrix} X_3 \\ X_1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 \\ 0 \\ -2 \\ 0 \end{bmatrix} \notin Basic\ feasible\ solution\ \text{(Infeasible solution)}$$

The third feasible solution

$X_B^3 = \begin{bmatrix} X_3 \\ X_2 \end{bmatrix}, C_B = [0 \quad 2] \quad , \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, B^{-1} = \frac{1}{|Adj\ B|} B^{\backslash} \quad ,$
$Adj\ B = 2$

$$a_{11} = 2\ , a_{12} = 0\ , a_{21} = -1\ , a_{22} = 1$$

$$B^{\backslash} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, B^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & -1/2 \\ 0 & 1/2 \end{bmatrix}$$

$$X_B^3 = \begin{bmatrix} X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 5/2 \end{bmatrix}$$

$$X_3 = 1/2\ , X_4 = 0\ , X_1 = 0\ , X_2 = 2.5$$

$$X = \begin{bmatrix} 0 \\ 02.5 \\ 0.5 \\ 0 \end{bmatrix} \in Basic\ feasible\ solution\ , Z_3 = 2 * 2.5 = 5$$

The 4th feasible solution

$X_B^4 = \begin{bmatrix} X_4 \\ X_1 \end{bmatrix}, C_B = [0 \quad 3], \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B^{-1} = \frac{1}{|Adj\ B|} B^{\backslash} \quad ,$
$Adj\ B = -1$

$$a_{11} = 1\ , a_{12} = -1\ , a_{21} = -1\ , a_{22} = 0$$

$$B^{\backslash} = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}, B^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}, B^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X_B^4 = \begin{bmatrix} X_4 \\ X_1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$X_3 = 0\ , X_4 = 2\ , X_1 = 3\ , X_2 = 0$$

$$X = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} \in Basic\ feasible\ solution\ \ , Z_4 = 3 * 3 = 9$$

The 5th feasible solution

$X_B^5 = \begin{bmatrix} X_4 \\ X_2 \end{bmatrix}, C_B = [0 \quad 2]\ ,$
$B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, B^{-1} = \frac{1}{|Adj\ B|} B^{\backslash}\ , Adj\ B = -1$

$$a_{11} = 2\ , a_{12} = -1\ , a_{21} = -1\ , a_{22} = 0$$

$$B^{\backslash} = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, B^{-1} = \frac{1}{-1} \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X_B^5 = \begin{bmatrix} X_4 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$X_3 = 0\ , X_4 = -1\ , X_1 = 0\ , X_2 = 3$$

$$X = \begin{bmatrix} 0 \\ 3 \\ 0 \\ -1 \end{bmatrix} \notin Basic\ feasible\ solution$$

The 6th feasible solution

$X_B^6 = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, C_B = [3 \quad 2] \quad , B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, B^{-1} = \frac{1}{|Adj\ B\ |} B^{\backslash} \quad , Adj\ B = 1$

$$a_{11} = 2 , a_{12} = -1 , a_{21} = -1 , a_{22} = 1$$

$$B^{\backslash} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, B^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X_B^6 = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$X_3 = 0 , X_4 = 0 , X_1 = 1 , X_2 = 2$

$$X = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \in Basic\ feasible\ solution$$

$$Z_6 = 3 * 1 + 2 * 2 = 7$$

The optimal solution $X = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \in Basic\ feasible\ solution$, $the\ opjective\ value\ Z = 7$

4.5.2 Optimization the Linear Programming Model

In the previous assumptions

$X_B = B^{-1}b\ ---(1)\quad basic\ feasible\ solution$

$z = C_B B^{-1} b ---(2)\ the\ value\ of\ objective\ function$

When$X_N = 0 \quad \Longrightarrow Non\ basic\ feasible\ solution$

Now let $X_N \neq 0 \quad \Longrightarrow \boldsymbol{N \equiv a_j}, \boldsymbol{X_N \equiv X_J}_{\Rightarrow J \in R\ Subset\ of\ A\ Matriex}$

Recompense those Assumptions in linear programming model

$$Z = [C_B \quad C_j] * \begin{bmatrix} X_B \\ X_j \end{bmatrix}$$

$$S.TO$$

$$[B \quad a_j] * \begin{bmatrix} X_B \\ X_j \end{bmatrix} = b$$

$$X \geq 0$$

Then

$$Z = C_B X_B + C_j X_j$$

$$S.TO$$

$$B X_B + a_j X_j = b$$

$$X \geq 0$$

$B X_B + a_j X_j = b \quad , B X_B = b - a_j X_j$

$X_B = B^{-1}b - B^{-1}a_j X_j ---(3)$

$Z = C_B X_B + C_j X_j \ , Z = C_B(B^{-1}b - B^{-1}a_j X_j) + C_j X_j$

$Z = C_B\ B^{-1}b - C_B B^{-1} a_j X_j \ + C_j X_j$

$Z = C_B\ B^{-1}b - (C_B B^{-1} a_j - C_j) X_j$

$Z = Z_1 - (Z_j - C_j) X_j ---(5)$

From equation (5) we say that there is an improvement in the solution according to the following

1. If the objective function (max) we say that there is an improvement in the solution
   If $(Z_j - C_j) < 0$
   $\theta\ or\ X_j = min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0 --$
   $-(6)\ point\ of\ improvement$
   $X_B = B^{-1}b - B^{-1}a_j X_j ---(7)\ improved\ soluton$
2. If the objective function (min) we say that there is an improvement in the solution
   If $(Z_j - C_j) > 0$
   $\theta\ or\ X_j = min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0 --$
   $-(6)\ point\ of\ improvement$
   $X_B = B^{-1}b - B^{-1}a_j X_j ---(7)\ improved\ soluton$

Example: find the optimal solution

$$max\ Z = 3X_1 + 2X_2$$

S.TO

$$X_1 + X_2 \leq 3$$

$$X_1 + 2X_2 \leq 5$$

$$X_1, X_2 \geq 0$$

Transforming a General LP into Equality Form

$$max\ Z = 3X_1 + 2X_2 + 0X_3 + 0X_4$$

S.TO

$$X_1 + X_2 + X_3 = 3$$

$$X_1 + 2X_2 + X_4 = 5$$

$$X_1, X_2, X_3, X_4 \geq 0$$

Then the matrix of this model

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}, C = [3 \quad 2 \quad 0 \quad 0]$$

$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$, $Let\ X_B = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix}$

$C_B = [0 \quad 0]$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$X_B = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\rightarrow X_3 = 3 , X_4 = 5 , X_1 = 0 , X_2 = 0$

$Z_1 = C_B B^{-1} b = [0 \quad 0]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

Now we search for optimization

The objective function (max) we say that there is an improvement in the solution

If $(Z_j - C_j) < 0$

$\theta\ or\ X_j = min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0 - - - (6)\ point\ of\ improvement$

$X_B = B^{-1}b - B^{-1}a_j X_j - - - (7)\ improved\ soluton$

If j=1 $\Longrightarrow (Z_1 - C_1) < 0$

$recompense\ in\ this\ equations\ C_B B^{-1} a_1 - C_1$

$[0 \quad 0]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 3 = -3 < 0$There is an improvement

If j = 2 $\Longrightarrow (Z_2 - C_2) < 0$

$C_B B^{-1} a_2 - C_2$

$[0 \quad 0]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2 = -2 < 0$There is an improvement

But if j=1 there is a best an improvement

x_1 is the internal value

$X_B = B^{-1}b - B^{-1}a_j X_j$

$\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} X_1$

To find the outside variable

$X_1 = \min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0$, $X_1 = \min \left\{\frac{3}{1}, \frac{5}{1}\right\}$

The min value $x_3 \Longrightarrow is\ outside\ variable$

$X_B{}^2 = \begin{bmatrix} X_1 \\ X_4 \end{bmatrix}$, $B\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $B^{-1} = \frac{1}{|AdjB|} B^{\backslash}$, Adj B = 1

$a_{11} = 1 , a_{12} = -1 , a_{21} = 0 , a_{22} = 1$, $B^{\backslash} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $B^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $X_B^2 = \begin{bmatrix} X_1 \\ X_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$Z_2 = C_B B^{-1} b = [3 \quad 0]\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = 9$

Now we search for optimization again

If j = 2 , $\Longrightarrow (Z_2 - C_2) < 0$

$C_B B^{-1} a_2 - C_2$

$[3 \quad 0]\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2 = 3 - 2 > 0\ There\ is\ no\ improvement$

If j = 3$\Longrightarrow (Z_3 - C_3) < 0$

$C_B B^{-1} a_3 - C_3$

$[3 \quad 0]\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 3 - 0 > 0$There is no improvement

The best optimal solution $X = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix}$, $Z = 9$

5. Computer algorithms

- Begin
- Define B, b, x_B, c_B
- Find $x_B = B^{-1}b$
- For j: u to v (u,v not exist in basic solution)
- Optim= $(C_B B^{-1} a_j - C_j)$
- If(optim<0) and (objective function = max)
- There is an improvement
- $X_j = min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0$
- $Z_j = C_B B^{-1} b$
- Else If(optim>0) and (ob_function = min)
- There is an improvement
- $X_j = min \left\{\frac{B^{-1}b}{B^{-1}a_j}\right\}, B^{-1}a_j > 0$
- $Z_j = C_B B^{-1} b$
- Else
- There is no improvement
- $Z_j = C_B B^{-1} b$

6. Conclusions

At the end of this paper there are found this results

- Computer algorithms help to find optimal solution with ease and ease.
- The Linear programming forces the administrator to be objective than decisions making on a personal basis.
- The possibility of the best use of the factors of production so that it can study all factors of production within the model.

ISSN 2229-5518

Recommendations: Linear programming is one of the mathematical models that help in decision-making. Therefore, the researcher recommends building an electronic application with graphical interfaces that makes it easier for administrative owners to process their data to make decisions.

7. References

[1] Blair,C. E., "The Iterative Step in the Linear Programming Algorithm of N. Karmarkar," Algorithmica, 1(4), pp. 537-539, 1986.

[2] Best,M. J., and K. Ritter, Linear Programming Active Set Analysis and Computer Programs, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1985.

[3] Bazaraa,M. S., H. D. Sherali, and C. M. Shetty, Nonlinear rogramming: Theory and Algorithms, John Wiley & Sons, Inc., third edition, 2006.

[4] Anstreicher,K. M., "A Monotonie Projective Algorithm for Fractional Linear Programming," Algorithmica, 1, pp. 483-498, 1986b.

[5] Adler,I., M. G. C. Resende, and G. Veiga, "An Implementation of Karmarkar's Algorithm for Linear Programming," Operations Research Center, Report 86-8, University of California at Berkeley, May 1986

[6] SMALE,S. 1983. On the average number of steps in the simplex method of linear programming, Mathematical Programming, 27, 241-262.

[7] DANTZIG ,G. B. 1992. An t-precise feasible solution to a linear program with a convexity constraint in l/t2 iterations independent of problem size, orking paper, Stanford University, Stanford, CA.

[8] TEO,C. 1996. Constructing approximation algorithms via linear programming relaxations: primal dual and randomized rounding techniques, Ph.D. thesis, Operations Research Center, M.LT. , Cambridge, MA.

[9] TSENG,P. 1989. A simple complexity proof for a polynomial-time linear programming algorithm, Operations Research Letters, 8, 155-159.

[10] TSUCHIYA,T . , and M. MURAMATSU. 1995. Global convergence of a long-step afne scaling algorithm for degenerate linear programming problems, SIAM Joural on Optimization, 5, 525-551.

[11] Gerald. Baillon, Applied Linear Programming Decision Aid Tool, op. cit., 1996,p08

[12] VANDERBEI, R. J., M . S. MEKETON, and B . A . FREEDMAN. 1986. A modifcation of Karmarkar's linear programming algorithm, Algorithmica, 1 ,395-407

[13] MICHELSimonnard Linear programming of economic calculus, dunod paris 1972. p09

[14] YE, Y., M. J. TODD, and S. MIZUNO. 1994. An O(FL)-iteration omogeneous and self-dual linear programming algorithm, Mathematics of perations Research, 19, 53-67.

[15] VANDERBEI, R. J . , J. C. LAGARIAS. 1990. I. I. Dikin's convergence result for the afne-scaling algorithm, in Mathematical Developments Arising fm Linear Programming, J. C. Lagarias and M. J. Todd (eds.) , American Mathematical Society, Providence, RI, Contemporry Mathematics, 1 14,109-119.

[16] YE, Y. 1991. An O(n3L) potential reduction algorithm for linearprogramming, MathematicalProgramming, 50, 239-258.

[17] AMOR.Farouk Linear Programming, Algiers, University Publications, 2000, p14.